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DISCUSSION PAPER

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1 Introduction

In this chapter, we review an approach to policy assessment that has been developed in the last two decades. This approach can be characterized by four features. The first feature is that it looks for evaluation criteria that are consistent with Pareto efficiency.[1] That means that individual preferences play a crucial role. Those preferences are assumed to be self-centered and to represent agents' well-informed and rational rankings of bundles of resources.

The second feature of the approach is that it follows the ethical objective of resource equality.[2] More precisely, it is only consistent with the view that some version of the Pigou-Dalton transfer principle is desirable, that is, social justice requires that resources be redistributed from richer to poorer agents. Which resources are relevant to give a precise meaning to this objective is a key question to which we return when we define the approach more precisely in the next section and when we present applications of the approach in Section 3.

The third feature of the approach is that, in addition to the information about the available resources in the economy, the only information about agents that is considered relevant is the information about the ordinal and non-comparable satisfaction of their preferences. That means that the only relevant information about agents consists of the sets of bundles of resources to which an agent is indifferent (the indifference curves, or indifference surfaces, or preference-equivalence classes) and how agents rank these sets. To be even more precise, that means that the information that is considered *not to be available* is a meaningful numerical representation of the preferences, according to which, for instance, there is the same utility difference between bundles $(2, 2, 2)$ and $(1, 1, 1)$ as between bundles $(3, 3, 3)$ and $(2, 2, 2)$, or the information that a given agent has the same satisfaction level with her current consumption as another agent with her own. Any information of either of these two kinds (measurability and comparability) is considered unavail-

[1] Throughout this chapter, we refer to the strong version of Pareto efficiency. If all agents weakly prefer an allocation over another one, then society also prefers the former. If, in addition, one agent strictly prefers the first allocation, then society also strictly prefers it.

[2] While the approach has mainly been developed in economic models in which the different dimensions are quantities of commodities, it can be applied to wider spaces, for instance when the different dimensions are functionings. See Fleurbaey and Maniquet, 2011a, Chapter 7.

able or irrelevant. As it will become clear in Section 2, *starting* with this limited information will not prevent us from *constructing* different ways of measuring well-being indices and comparing well-being levels across agents. What is thus key in the SOF approach is that no information of this kind is assumed from the outset.

One motivation for restricting the analysis to that information is the view that human beings are morally autonomous agents, so that their choices should be respected and how far they go in the fulfillment of their aspiration or how successful they are in the accomplishment of what they consider a good life is irrelevant for the allocation of resources. This motivation implies that the only legitimate reason to use information about individual satisfaction levels comes from the objective of taking Pareto efficiency into account. It is well known that ordinal and non-comparable satisfaction levels are all we need to apply the Pareto criterion. As a consequence, the SOF approach limits itself to using ordinal and non-comparable information on preferences in order to combine efficiency and resource equality views (see also Fleurbaey, chapter , this Handbook, for another derivation of ordinalism).

Another legitimate motivation for restricting one's attention to preference satisfaction is the practical difficulty to extract more information. It is well-known that choices only reveal information of that nature and obtaining more information may raise unsolvable manipulation issues (information about preferences can also be drawn from different kinds of expressed preferences, among which satisfaction surveys. See Decancq and Neumann, chapter , this Hanbook).

The fourth feature of the approach is that the criteria that are built are complete orderings of the set of relevant allocations of resources. Such orderings are called Social Ordering Functions (SOF). Social orderings are typically built as a function of the relevant parameters of the problem at hand. The word "Social" refers to the fact that the objects that are ranked are social allocations, that is, lists of bundles for all members of the society. The word "Ordering" refers to the fact that the objective is simply to rank those lists and not (necessarily) to construct a notion of social "Welfare."

To sum up, a fair SOF is a formal object that associates to each economy in an admissible domain of economies a complete ordering of the relevant set of allocations. Such orderings are required to be consistent with the Pareto criterion and to satisfy some version of the Pigou-Dalton resource transfer principle. The admissible argument of a SOF is the information about the resources available in the economy as well as the ordinal and non-comparable individual preferences. Fleurbaey and Maniquet (2011a) provide an extensive description of the literature on SOF's. This chapter relies heavily on that book, which will serve as a general bibliographic reference for this chapter.

The SOF approach is characterized by the four features outlined above, but it shares each feature with many other approaches in welfare economics, including, of course, approaches reviewed in this Handbook. All approaches look for criteria that are compatible with the Pareto criterion, except the multidimensional social indices approach reviewed by Chakravarty and Lugo, chapter , this Handbook. That approach, on the other hand, also respects the objective of resource equality, an objective that is shared by the theory of fair allocation. The main difference between the theory of SOF's and that of fair allocation is that the latter does not look for complete rankings of the possible allocations. It limits itself to identifying the set of best allocations. We return to the theory of fair allocation in Section 4. Looking for complete orderings of the allocations, however, is standard in welfare economics, since Bergson, Samuelson and Arrow. Welfarism also provides us with complete orderings. Cost-benefit analysis, reviewed by Boadway, chapter , this Handbook, is also able to provide complete and transitive criteria. On the other hand, those criteria are not based on any reference to the objective of resource equality. Finally, limiting the relevant information about agents to ordinal and non-comparable preferences is also standard, although welfarism typically aggregates welfare levels that have a cardinal meaning and/or that are interpersonally comparable. We also return to welfarism in Section 4.

In Section 2, we describe the main achievements of the SOF approach. In Section 3, we present two applications, which show how SOF's can be used to evaluate policies. The first application concerns labor income taxation. The second application concerns the measurement of poverty. In Section 4, we discuss the relationship between the SOF approach and some other approaches to the construction of criteria to evaluate policies.

2 Fair Social Ordering Functions

In this section, we present the main lessons that are to be drawn from the literature on SOF's. The first lesson is about the relationship between Pareto efficiency and resource equality. The second lesson is about the way SOF's aggregate satisfaction levels. The third lesson is about the diversity of SOF's that turn out to be justified.

The SOF approach aims at providing policy evaluation criteria. Those criteria are formally derived by imposing desirable requirements, axioms, to SOF's. The requirements combine Pareto efficiency with the objective of resource equality. It is transparent that these two objectives are conflicting. Indeed, equally dividing all resources that are available in society obviously meets the latter objective, but it is typically Pareto inefficient when agents have different preferences.

Pareto efficiency forces us to evaluate policies on the basis of their effect on preference satisfaction, but equality of resources forces us to carefully look at the different components of the bundle of goods consumed by each agent. The conflict between the two objectives has been studied since at least Blackorby and Donaldson (1988) and has been given its most general formulation by Fleurbaey and Trannoy (2003). These authors show that a SOF satisfying the Pareto criterion fails to be quasi-concave in commodities. To put it differently, if a ranking of allocations satisfy the Pareto criterion, then there must exist a situation in which one agent consumes strictly more of all goods than another agent and transferring even small quantities of goods from the former to the latter agent leads to a worse allocation in terms of the social ranking.

This conflict forces us to give priority either to the Pareto criterion or to resource equality. One can see the theory of multidimensional inequality measurement as the theory that can be developed if we give priority to the objective of resource equality and drop the Pareto criterion. The SOF approach, on the contrary, gives priority to the Pareto criterion. The first lesson that can be drawn from the literature, consequently, is that the objective of resource equality has to be amended in ways that are compatible with the Pareto criterion. An example of such amendment is the requirement that the objective of resource equality holds at least among agents identical in all dimensions, including their preferences. This is a weak requirement and it is satisfied by all the SOF's that are proposed in the literature.

Other resource equality requirements, more ambitious than the restriction to identical agents, have been studied. They typically depend on the particular resource allocation problem one is studying. If the problem is one of the division of unproduced goods, then equal split of the goods may play a particular role. One may require that the objective of resource equality holds between an agent who consumes more than equal split and an agent who consumes less (see Fleurbaey and Maniquet, 2008). If production takes place and agents privately contribute to the production, by providing labor time for instance, then the reference to the laisser-faire allocation in economies where all agents have the same production skills may help us define an objective of resource equality that is also compatible with the Pareto criterion (see Fleurbaey and Maniquet, 2006).

Many SOF's can be defined, in different environments, on the basis of the Pareto criterion and amendments to the objective of resource equality. To select among those many SOF's, robustness requirements are added. Robustness requirements follow two ideas. The first idea is that a good SOF should be simple, that is, the social ranking between two allocations should only depend on a limited amount of information. The second, related, idea is that the social ranking in a particular problem should be similar to the social ranking in a similar problem.

There are several sets of robustness requirements. For the sake of this presentation, it is useful to give some details about one of them. One may require, specifically, that the social ranking between two allocations depend on a limited amount of information about the agents' preferences. This is reminiscent of the celebrated Arrovian Independence property. Arrow (1963) studied the consequence of requiring that the social ranking towards two allocations only depend on agents' preferences over those two allocations. It is well known that this requirement, combined with the Pareto criterion, leads to dictatorial social orderings. The following weakening of Independence, proposed by Hansson (1973), is sufficient to escape from Arrow's dictatorship result and select among social orderings satisfying the Pareto criterion and resource equality requirements. It requires that the social ordering between two allocations only depend on the agents' indifference curves containing the bundles they are assigned in these two allocations. In terms of choices, that requirement can be defined as follows. If an agent's preferences change in such a way that all binary choices involving the bundles obtained by that agent in two allocations remain unchanged, then the social ordering towards these two allocations remains unchanged as well.

Imposing the Pareto criterion, fairness properties and this independence requirement turn out to have a further, unexpected, consequence. Several scholarly contributions have proven, indeed, that the combination of these three requirements imply that the only acceptable SOF's are of the maximin type (this scholarship is reviewed in Fleurbaey and Maniquet, 2011a, Chapter 3). That is, any SOF satisfying the three types of requirements orders allocations in the following way: first, a well-being index is built to measure the agents' satisfaction levels, and, second, the individual well-being levels are aggregated using a maximin aggregator, which means that the primary objective is to maximize the minimal well-being level.[3]

The reasoning why the aggregator needs to try to maximize the minimal well-being level goes as follows. The Pareto criterion forces us to aggregate satisfaction levels, which means that not too much attention can be paid to the bundles of goods consumed by the agents. On the other hand, the fairness objective forces us to look at what agents consume, as we would like to equalize assigned resources. The typical fairness requirement is that a progressive transfer from an agent consuming strictly more of all the goods than another agent to the latter should be a social improvement, at least when these two agents have the same preferences. Combining the Pareto criterion with this transfer requirement turns out to be very demanding. This is typically the case when the shape of the preferences of the agents is such that the value for them of a given transfer depends on the bundle they originally consume when they receive that transfer. It can be shown that a transfer of a given amount of resources from a relatively richer agent to a relatively poorer agent may be equivalent, in terms of preferences satisfaction, to the loss of an arbitrarily large amount of resources by the richer agent combined with the gain of an arbitrarily small amount of resources by the relatively poor one.

Let us, for instance, consider the balanced transfer of \$ 100 between two agents working half time. If income effects are large and depend on the satisfaction levels of the agents, it can perfectly well be the case that loosing \$ 100 for the richer agent when she works half time is equivalent, in terms of preference satisfaction, to loosing a much larger amount, say \$ 500, when she works full time, and, at the same time, gaining \$ 100 for the poorer agent when she works half time is equivalent to gaining a much lower amount, say \$ 50, when working full time. If the transfer when they both work half time is viewed as a social improvement, then the following leaky bucket transfer should also be viewed as a social improvement when they both work full time: take \$500 away from the pocket of the richer agent, and only give \$ 50 to the poorer agent.

Adding the requirement that the social ordering function should not de-

[3]Here we call maximin all the aggregators that begin by strictly preferring the allocation in which the lowest well-being level is the highest, independently of how they break ties when the lowest well-being levels are identical in two allocations. This is why we sometimes speak of the maximin family of aggregators. The leximin aggregator, defined below, is the most prominent example of a maximin aggregator, but many others can be thought of.

pend on changes in preferences that do not affect the indifference surface of the agents at the contemplated allocations does not allow us to rank allocations as a function of the different values one given transfer of resources may have for an agent. As a consequence, transfers are a social improvement only if we use a maximin aggregator. Going back to our example, it means that any leaky bucket transfer between two agents can be decomposed into a series of other, smaller, leaky bucket transfers that all are equivalent, in terms of preference satisfaction, to balanced transfers. The fact that robustness properties do not allow us to check whether a leaky bucket transfer can indeed be decomposed into a series of transfers having that property forces us to do as if such a decomposition was possible, with the outcome that any arbitrarily small improvement for a poorer agent leads to an allocation that is higher in the social ranking, however large the loss of the richer agent.

As a matter of fact, other robustness requirements that are not independence requirements with respect to changes in preferences also lead to SOF's of the maximin type. It is, therefore, the second lesson of the literature that admissible SOF's must aggregate well-being levels in a maximin way, when the requirements we have presented above are imposed.

The application of the SOF approach to labor income taxation below will illustrate the practical advantages of having to maximize social rankings of the maximin type. The maximin results of the SOF literature, however, should not hide the large variety of SOF's that enter the maximin family. The third lesson of the literature, indeed, is that many indices of well-being can be justified by different combinations of Pareto, resource equality and robustness requirements. We illustrate this variety in the simple pure fair division model, following Fleurbaey (2005) and Fleurbaey and Maniquet (2008).

Let us assume that there are L goods, and each entry in the vector $\Omega \in \mathbb{R}^L_{++}$ is the total amount available of one such good. As a result, Ω contains all the relevant information about the resources available in that economy, and these resources have to be divided among agents having equal rights on them. Agents have continuous, monotonic and convex preferences. Preferences may differ across agents. There are two main SOF's that can be used in order to allocate goods among the agents.

The first SOF relies on the concept of "egalitarian-equivalence" and applies the leximin criterion to specific individual well-being indices. The leximin criterion consists in applying the maximin criterion lexicographically: first, prefer the allocation with the largest minimal well-being level, then, in case of a tie, prefer the allocation with the largest second lowest one, etc. The specific individual well-being levels to which the leximin criterion is applied by the SOF are defined as follows. They evaluate every agent's bundle by the fraction of Ω to which this agent is indifferent.

The concept of "egalitarian-equivalence" has a long tradition in welfare economics (see Fleurbaey and Maniquet, 2011a, Chapter 1, for details). The specific SOF we are interested in here was introduced by Pazner and Schmeidler (1978). It is illustrated in Figure 1. There are two agents. Their preferences, denoted R_1 and R_2, are represented by two indifference curves. We have to order two allocations, (x_1, x_2) and (x'_1, x'_2). Agent 1 is indifferent between consuming the bundle of goods x_1 and a fraction λ_1 of the resources to be allocated. We compute λ_2, λ'_1 and λ'_2 the same way. We are thus left with comparing the vectors (λ_1, λ_2) and (λ'_1, λ'_2) by using the leximin. As λ'_2, the minimal well-being level at (x'_1, x'_2), is larger than λ_1, the minimal well-being level at (x_1, x_2), we conclude that allocation (x'_1, x'_2) is socially preferred to (x_1, x_2).

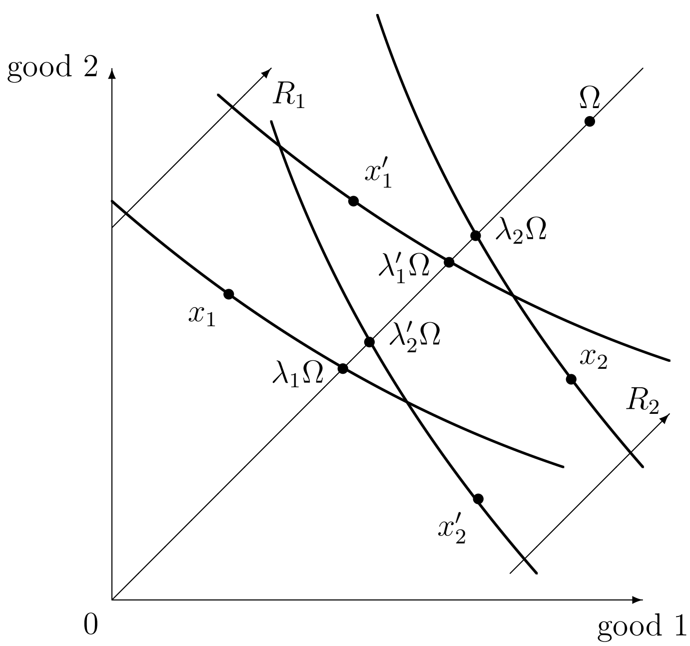

Figure 1: Illustration of an egalitarian equivalent SOF. Allocation (x'_1, x'_2) is socially preferred to (x_1, x_2).

The second SOF we present here has a Walrasian flavor. It relies on the maximin criterion applied to money-metric utilities (for a complete discussion of money-metric utility, see Fleurbaey, chapter , this Handbook), but with the peculiar feature that the reference price that serves to measure such utilities is specific to every allocation, and is computed so as to obtain the most favorable maximin evaluation for the contemplated allocation. It is illustrated in Figure 2. One allocation, (x_1, x_2, x_3), is represented, together with the indifference curves of the three agents through their bundles. A line is tangent to the indifference curves of agents 1 and 2. That line corresponds to a budget line at price p. This allows us to compute what can be called the equivalent endowments of the agents. The equivalent endowments of both agents 1 and 2, at that price vector, can be measured by the fraction $\lambda_1 = \lambda_2$ of the total social endowment. For each such price vector, by tangency with the indifference curves, an individual equivalent endowment can be measured for each agent. Let us observe that price p is the precise vector that maximizes the minimal such endowment, when they are measured in fractions of Ω. Vector p can also be used to measure the equivalent endowment of agent 3, λ_3, in a similar way, by tangency with her indifference curve. The resulting endowment would be larger than that of agent 1 or 2. In conclusion, choosing the vector p that maximizes the minimal equivalent endowment gives us a way to transform an allocation into a list of individual numbers $(\lambda_1, \lambda_2, \lambda_3)$. The SOF we are interested in here works by applying the leximin criterion to those lists of numbers.

Both SOF's presented above satisfy the Pareto criterion. They also satisfy similar robustness properties. The main difference between them lies in the resource equality requirement each of them meets. The first SOF satisfies the requirement that a redistribution from an agent consuming a larger bundle than the equal-split share of the endowment to an agent consuming less is a social improvement. The second SOF satisfies the requirement that the allocations that are considered socially best among the feasible allocations are no-envy allocations: no agent strictly prefers the bundle assigned to another agent to her own bundle.

Those two resource equality requirements are relevant for the division problem of a social endowment of goods. The literature on SOF's has shown that different allocation problems could involve different resource equality requirements, which, in turn, combined with Pareto and robustness requirements, would justify different SOF's. The research agenda of that literature consists first in studying resource equality requirements in different allocation problems to characterize relevant SOF's and then to apply those SOF's to the evaluation of policies, as we exemplify in Section 3 below. Let us con-

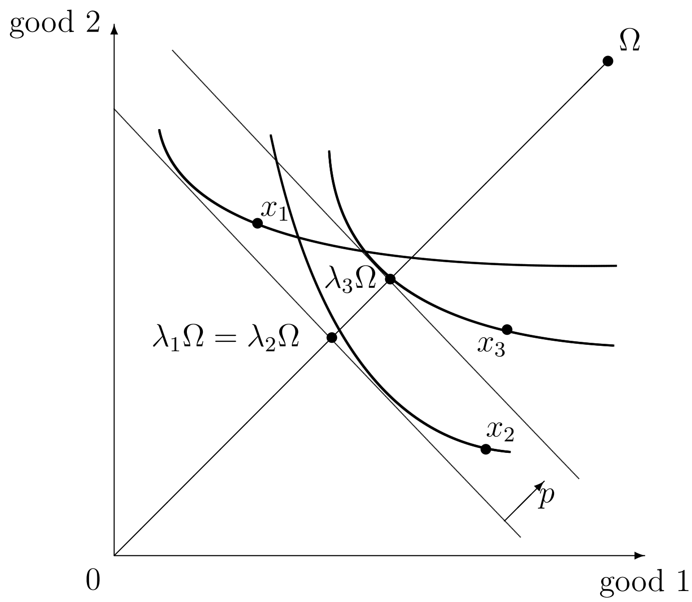

Figure 2: Illustration of a Walrasian SOF. Allocation (x_1, x_2, x_3) is transformed into $(\lambda_1, \lambda_2, \lambda_3)$. The leximin aggregator is applied to those numbers.

clude this section by noting that the allocation problems to which the SOF approach has been applied is extending, and it includes production of public goods problems (see Maniquet and Sprumont, 2004, 2005, Fleurbaey and Sprumont, 2009), allocation of indivisible objects (Maniquet, 2008), the pure compensation problem (Valletta, 2009), the allocation problem with unequal production skills (Fleurbaey and Maniquet, 2005, 2006, 2007, 2011b), and the allocation problem when agents have unequal health statuses (Fleurbaey, 2005b, Valletta, 2012, and Fleurbaey and Valletta, 2014).

The two SOF's presented above also illustrate how the SOF approach allows us to construct comparable well-being indices from non-comparable preferences. The information that is given at the beginning consists of the map of indifference curves of each agent. Nothing is available to compare them. The requirements that are imposed on the social rankings can only be satisfied if allocations are compared on the basis of the leximin criterion applied to those vectors of λ's. They can be viewed as well-being indices, and, crucially, comparable ones. In Figure 1, for instance, it becomes meaningful to claim that agent 1 consuming bundle x_1 is worse-off than agent 2 consuming x'_2. It is also a consequence of the requirements we impose that those well-being indices remain ordinal. Because we have to apply the leximin aggregator, the difference in well-being between agent 1 consuming x_1 and x'_1 is meaningless. It is not needed to rank allocations according to the efficiency, resource equality and robustness requirements that characterize that SOF.

3 Applications

In this section, we present two policy evaluations that are based on SOF's. The first application bears on the design of a reform of the labor income tax system. The second application bears on the measurement of poverty. As we will see, a key difference between the two applications is that the first one does not require to estimate individual preferences (it is sufficient to know that some specific preferences exist in society), whereas the second one does.

3.1 A reform of the labor income tax system

Labor income tax systems typically require that taxpayers pay an amount that only depends on their gross income. This excludes computing taxes as a function of skill or labor time. This is justified by the fact that gross incomes are typically easier to observe (or less easy to manipulate) than skills or labor time. It is the typical assumption in the optimal tax literature since Mirrlees (1971).

A taxpayer, as a result, faces a tax function that determines how much tax she will have to pay as a function of her gross income. She chooses how much gross income to earn as a function of that tax function.

Optimal tax theory is concerned with the design of such tax function under the objective of maximizing an ethical objective, which is represented by a social welfare function. The typical assumption is that the planner only knows the distribution of skills and preferences in the population but is unable to identify the skill and preferences of any given agent.

SOF's can be used to evaluate tax functions (see Fleurbaey and Maniquet, 2006, 2007, 2011a, 2011b). This is what we illustrate in this section. We begin by defining one specific SOF that was characterized in the literature (see Fleurbaey and Maniquet, 2011a and 2011b). Then we show how to use it to evaluate a tax function.

This SOF works by applying the leximin criterion to the following well-being measure, inspired by Kolm (1996): the well-being of an agent is equal to the income that agent would earn by working half time and by being paid at her actual skill in budgetary conditions that leave her indifferent to her actual situation. This well-being measure bears some similarity with the second example in the previous section. It is illustrated in Figure 3. The relevant consumption set is composed of two goods, labor time, measured horizontally, and the consumption of some good (or income), measured vertically. An agent is represented, consuming bundle x_i. Her preferences, R_i, are illustrated via the indifference curve through x_i. This agent is indifferent between consuming x_i and being free to choose her labor time in the budget set that is represented in the figure. Let us call it the equivalent budget of agent i at x_i. The slope of this budget set, mentioned in parentheses below the budget frontier, is equal to the skill of that agent, s_i. This is how the skill influences the measurement of the well-being index of that agent, and this is why that well-being measure exhibits some Walrasian flavor. Finally, the well-being level of that agent at x_i, denoted w_i, is equal to the income associated with a labor time of 0.5 at the equivalent budget.

The question can be stated as follows: given a tax function, how can we measure the well-being of relevant agents, and how can we go from the measurement of well-being to the evaluation of the tax function?

The measurement of well-being based on the tax function is illustrated in Figure 4. The C function determines consumption (that is, after-tax income) as a function of gross (that is, pre-tax) income. The 45° line, with a slope of 1 mentioned below it, corresponds to the C function that would prevail in the absence of taxation. Indeed, if agents are paid according to their skills and no tax is collected, then any additional unit of gross income leads to an identical additional unit of consumption.

Let us fix a gross income level, say y. What can be said about the well-being of agents earning precisely y? Something precise can be said under the assumption that at least one agent earning y, let us call her i, works full time, that is, has a labor time $\ell_i = 1$. The choice of a full time job reveals, first, that her skill s_i is equal to y, and, second, that this agent prefers y and the corresponding consumption level $C(y)$ over all other $y' < y$ and their corresponding consumption. This is sufficient to draw the equivalent budget of slope s_i to which this agent is indifferent. This budget is drawn in the

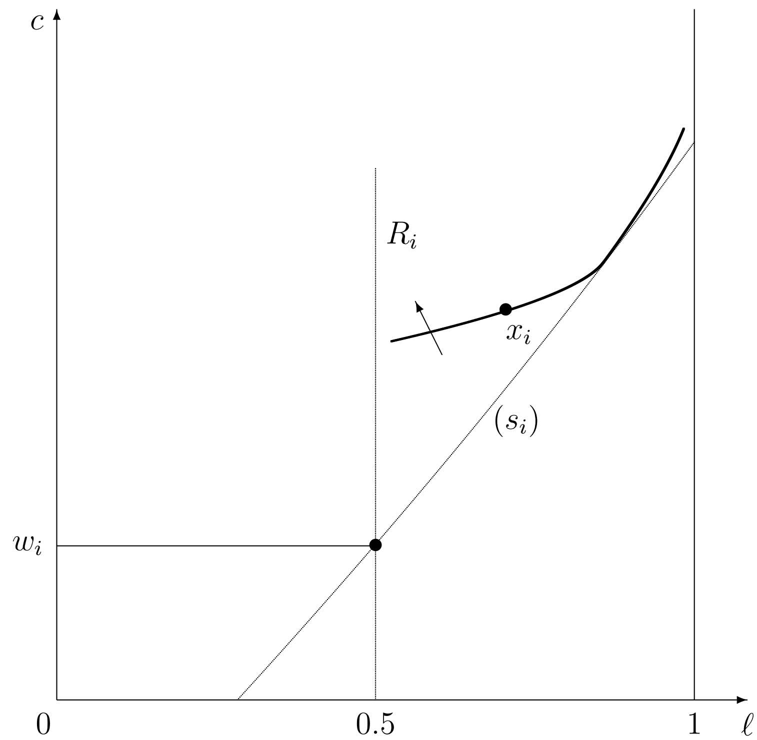

Figure 3: Illustration of a well-being measure that depends on the agent's skill: the well-being of an agent with preferences R_i consuming bundle x_i is equal to w_i.

figure. It is the 45° line ending at $(y, C(y))$. Indeed, let us observe that facing such a budget of slope s_i (remember that $s_i = y$), this agent would choose to work full time. The agent is therefore indifferent between her actual bundle $(y, C(y))$ and being free to choose in that budget of slope s_i. Once this equivalent budget has been identified, it is not difficult to measure the well-being of that agent. Indeed, should this agent work half time, she would have earned w_i. This is her well-being measure.

Something less precise but extremely useful can be said about all agents have the same skill, y, but having chosen to work less. If such an agent has chosen to earn an income of, say, y'', then she reveals that she weakly prefers the bundle $(y'', C(y''))$ to all available bundles, including $(y, C(y))$. As a result, even if we cannot identify their equivalent budget precisely, we know that they are larger (with respect to inclusion) than that of the agent

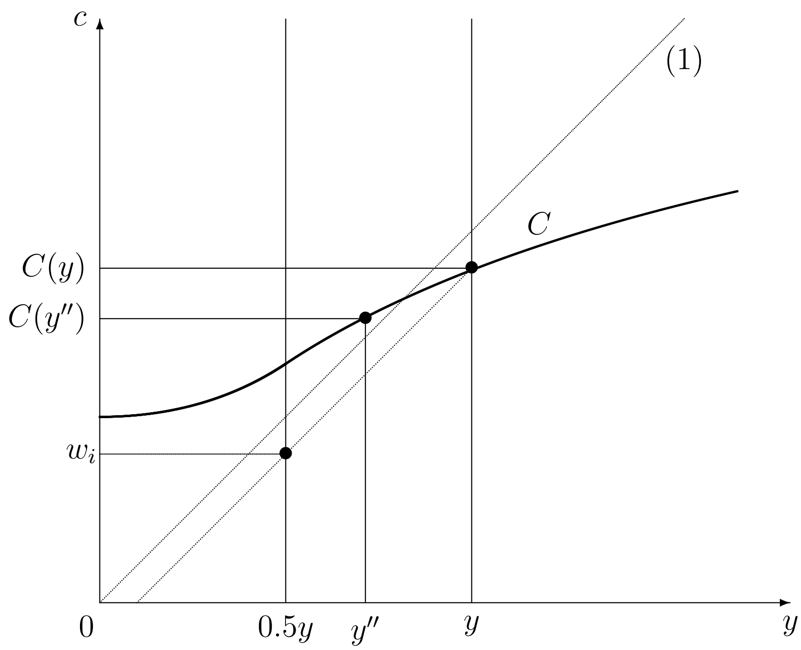

Figure 4: Evaluation of a tax function: computing the minimal well-being level, w_i, among agents having a skill level equal to y.

consuming $(y, C(y))$. This proves that the well-being of all agents having a skill equal to y is equal to w_i or larger.

Now, let us look at the consequence of aggregating well-being levels by using the leximin criterion. The imprecise information that we gathered so far is all what we need. Indeed, the only relevant agent, among all agents with a skill level of y, is the agent working full time, and we have been able to measure her well-being precisely. It remains to iterate the procedure so as to measure the minimal well-being level at all skill levels. By doing so, the worst-off agent in the population can be identified. More precisely, the gross income earned by the agent that turns out to be the worst off in the economy can be precisely identified. Observe that she need not be a low-skill agent. The reform that needs to be implemented is a decrease in the tax paid (or an increase in the subsidy obtained) by agents earning that gross income.

If the policy assessment question is that of choosing between two tax systems, then it is sufficient to apply the reasoning above to each one of them. Once the well-being level of the worst off in each system has been identified, it remains to conclude that the system with the largest minimal well-being level is better than the other.

The reasoning above has allowed us to present the way to evaluate a tax system in order to identify the reform that needs to be implemented to obtain allocations higher in the social ranking. This application shows that using a maximin aggregator is a considerable advantage when one aggregates well-being measures based on fairness principles and one can only rely on limited information due to the incentive constraints.

3.2 Poverty measurement

Here, we present an application of egalitarian equivalence to the measurement of poverty. The theory of poverty measurement, as pioneered by Sen (1976), considers a vector of individual incomes and a poverty line, that is, an income threshold under which an agent qualifies as poor. The quantity of poverty of an economy is computed as a function of the vector of incomes below the poverty line.

Measuring poverty from an income vector is justified only if incomes are considered as a good proxy of individual well-being (see Cowell, Chapter , this Handbook). There are many reasons, however, why income is a poor well-being measure. First, there are numerous relevant well-being dimensions that are not marketed. Health is the typical example. Second, there are many market imperfections, such as transaction costs, discrimination, price rigidities, etc, which prevent people from freely choosing in their budget. Third, different people may face different prices, depending on where or when they live, and this fact alone may prevent incomes from being an interpersonally comparable way of measuring well-being.

A solution to these problems consists in measuring poverty from a multi-dimensional perspective. Agents are no longer characterized by their income but by a complete list of their achievements in dimensions that are relevant for their well-being. The difficult question of how to aggregate these many dimensions has recently given rise to a large number of contributions. In line with the SOF approach we present in this chapter, one proposal is to use agents' preferences themselves, and to acknowledge heterogeneity in preferences. This is the avenue that Decancq, Fleurbaey and Maniquet (2014) have explored. They have proposed an axiomatic study of multidimensional poverty measurement based on preferences. Their axiomatic study imposes a strong decomposition requirement on the poverty measure. As a consequence, their solution is almost equivalent to an axiomatic study of the measurement of well-being (individual well-being being simply the opposite of individual poverty), and it bears close relationship with the study of well-being measures developed in Fleurbaey and Tadenuma (2014) and Fleurbaey and Maniquet (2014).

The axiomatic analysis of Decancq, Fleurbaey and Maniquet (2014) does not start with the definition of a poverty threshold in all dimensions. On the contrary, it is only assumed that a threshold exists in terms of a minimal level of satisfaction of the individual preferences. As a consequence of the axioms, though, the authors show that the only way of measuring poverty precisely consists in imposing a common minimal consumption bundle. An agent qualifies as poor if she consumes a bundle she deems worse than the common minimal bundle. An agent qualifies as non-poor if she consumes a bundle she deems better than the common minimal bundle. Individual poverty is then measured by using egalitarian equivalence along the ray from the zero bundle (the worst bundle for each agent) to the poverty line bundle. This is slightly different from the Pazner-Schmeidler egalitarian equivalent SOF presented in Section 2. There, individual well-being levels are measured in terms of fractions of the social endowment. Here, the key reference point is the poverty line bundle, and it must be related to the ethical choice of the policy maker who fixes a minimal consumption in each of the relevant dimensions. The poverty index one obtains, though, is formally a SOF. In Decancq, Fleurbaey and Maniquet (2014), only a weak resource equality requirement is imposed. As a consequence, the resulting SOF is not of the maximin type. On the contrary, any inequality averse aggregator is acceptable.

Measuring poverty by using such a SOF requires to estimate preferences. This is the authors' strategy. Their strategy consists in using satisfaction data, and taking satisfaction reports as a proxy for the degree of satisfaction of the preferences (for a related but different use of satisfaction data for policy evaluation, see Dolan and Fujiwara, chapter , this Handbook). Introducing fixed effects in the regression allows them to get rid of many non-ordinal aspects of the satisfaction reports.

4 Comparison with related approaches

4.1 Bergson-Samuelson social welfare function

Let us begin by noting that SOF's like the ones we present in this chapter are Bergson-Samuelson social welfare functions (see Bergson, 1938, Samuelson, 1947, and Weymark, chapter , this Handbook). The two authors insisted that their functions could use ordinal and non-comparable information on preference satisfaction as arguments (for a detailed discussion of the nature of Bergson-Samuelson social welfare functions, see Fleurbaey and Mongin, 2005). This is exactly what is done here. Social orderings as we define them here use ordinal and non-comparable preferences as arguments, and they satisfy the Pareto criterion.

Bergson-Samuelson social welfare functions can be viewed as more general than SOF's in three ways. First, the well-being levels that they aggregate need not be only based on ordinal and non-comparable information on preference satisfaction. The literature in welfare economics has even stressed the possibility of making interpersonal comparison. Second, the aggregator need not be in the maximin family.

What makes Bergson-Samuelson social welfare functions different from Arrovian social welfare functions is that the former are not required to satisfy cross-economies robustness properties. The third reason why our SOF's are more specific than Bergson-Samuelson social welfare functions is that, as we explained in Section 2 above, SOF's do not satisfy the Arrovian Independence property but do satisfy some weakening of it.

In conclusion, one may see the SOF approach as a systematic study of Bergson-Samuelson social welfare functions, in which the emphasis is put on ordinal and non-comparable individual preferences, on the objective of resource equality, and on weakenings of Arrow's Independence requirement that are compatible with resource equality.

4.2 Welfarism

Welfarism is the normative stance that is based on the following two principles. First, welfare is an independent normative concept from which individual well-being measures can be derived. Second, only well-being levels matter for ethical judgments (detailed surveys are provided, among others, by Bossert and Weymark, 2004, and d'Aspremont and Gevers, 2002; a discussion of the relationship between Bergson-Samuelson social welfare functions and welfarist social welfare functions is provided by Weymark, Chapter , this Handbook). Welfarists, however, differ in their interpretation of what notion of welfare is relevant for the aggregation (for detailed discussion of different possible notions of well-being, see Bykvist, chapter , Haybron, chapter , and Hurka, chapter , this Handbook).

Two main notions of welfare are influential in economics. The first notion was popularized by Harsanyi (1953), Vickrey (1945), Mirrlees (1982), among many others. They consider that welfare is a real, subjective, measurable and comparable object. This is in clear contrast with the well-being functions that turn out being embedded in the SOF's we are interested in. The second notion is that welfare is a constructed object that reflects the ethical views of the policy maker about the determinants of the measurability and the comparability of well-being among agents.

Welfarism has received strong axiomatic justifications from Sen's theory of Social Welfare Functionals. That theory studies how rankings of allocations can be obtained from requirements of independence of transformation of welfare levels that precisely capture assumptions on the measurability and comparability of welfare. Utilitarianism, for instance, is closely related to the requirement that social rankings be independent of affine transformations of welfare levels with a common transformation of the units of measurement (the so-called cardinality and unit-comparability requirement). Leximin, on the other hand, is closely related to the requirement that social rankings be independent of any common strictly increasing transformation of the welfare levels (the so-called co-ordinality requirement; see, for instance, d'Aspremont and Gevers, 2002, for a survey).

To study the relationship between SOF's and welfarism, we need to distinguish between the nature of the social rankings that are obtained and the way to justify them. In terms of social rankings, one can claim that the SOF's that are prominent in the SOF approach are welfarist, as long as we stick to the second notion of welfare described above, that is, the notion that welfare indices are constructed objects that reflect the ethical views of the policy maker. The egalitarian equivalent SOF presented in Section 2 above, for instance, is welfarist in the sense that it uses a welfarist aggregator, the leximin, to individual well-being indices, that is, the indices that are constructed by indifference to fractions of the social endowment. These well-being indices are fully comparable across agents. Even if the analysis starts with non-comparable preferences, at the end we obtain the way to compare the well-being gains of some agents with the losses of some others. Such comparisons are, of course, necessary to evaluate policies. The SOF approach provides us with a theory that deduces the comparisons among agents from more fundamental requirements, especially resource equality requirements.

In terms of the way to justify social rankings, on the other hand, the SOF approach is fundamentally non-welfarist. Indeed, requirements of independence of transformation of welfare levels are foreign to the SOF approach. To some extent, the fact that SOF's look like welfarist social welfare functions is an accident. It simply turns out that imposing Pareto efficiency, resource equality and robustness requirements forces us 1) to construct well-being measures based on the bundles of commodities or functioning's that are consumed by the agents and 2) to maximize the lowest well-being level, but this way of ranking allocations does not come from the analysis of which transformation of welfare levels an index should be independent of.

4.3 The theory of fair allocation

The question of how to allocate resources fairly has originally been raised in terms of fair allocation rules. In the resulting theory of fair allocation, only the best allocations are identified (see Thomson, 2011, and chapter , this Handbook, for a survey). That is, the objective is to justify allocation rules, which identify the Pareto efficient and fair allocations in each economy from a domain of economies. Contrary to the SOF approach, the objective is not to rank the other allocations.

Fair allocation rules have been justified in many different contexts. It is impossible to give a detailed overview in a few lines. What is important for our purpose in this chapter is that two rules qualify as the most prominent fair allocation rules, when the issue is to deal with heterogenous preferences. The first rule is the following egalitarian equivalent allocation rule. It selects all Pareto efficient allocations having the property that each agent is indifferent between her assigned bundle and a common fraction of the social endowment. The second allocation rule is the equal income Walrasian allocation rules. It selects the set of allocations that are competitive equilibrium allocation after the total resources have been divided equally among all agents. At a competitive equilibrium allocation, each agent maximizes her preferences over a set of opportunities that is the same for all agents (see Kolm, 1968, for the first axiomatic study of that rule).

The relationship between those two allocation rules and the two SOF's presented in Section 2 should be clear: each allocation rule selects the allocations that are considered socially best for the corresponding SOF. The theories of allocation rules and SOF's have a lot in common, in spite of the former being much more developed than the latter. The main difference, though, lies in the fact that an allocation rule is silent about suboptimal allocations. The two applications we presented in the previous section, we think, have illustrated the advantage of the SOF approach. Indeed, an allocation rule would have nothing to say in the evaluation of a tax system, for instance, if the question is to identify the share of the tax scheme that should be improved, if only small changes can be implemented.

5 Conclusion

In this chapter, we have presented the SOF approach to policy assessment. We have shown how evaluation criteria can be constructed by a combination of Pareto, resource equality and robustness requirements. We have also shown how such criteria can be employed, either by using an argument involving incentive compatibility considerations, or following an estimation of individual preferences.

The SOF approach is normatively attractive provided one adheres to two fundamental ethical choices. The first choice is that people's choices should be respected. That is implemented by the use of preferences as a basic ingredient of the analysis and the requirement that the Pareto criterion be satisfied. The second choice is that economic justice is a matter of fairness, that is, of equality in the bundles of goods or functioning's that agents have access to. Once this choice is made, one still has the freedom to choose among the many resource equality requirements that can be imposed, as illustrated above.

A secondary ethical choice that is also implied by the SOF approach is that preferences are of ordinal and non-comparable nature. Roemer (1998) is an example of a study of fairness in which individual well-being indices are assumed to be cardinal and comparable. In the SOF approach, on the contrary, individual preference satisfaction levels are assumed to be ordinal and non-comparable. As a matter of fact, constructing SOF's, as illustrated above, boils down to constructing a way to cardinalize and/or build comparisons among satisfaction levels of different agents, but there is a key difference between assuming comparability or constructing it on the basis of Pareto, resource equality and robustness requirements.

The fourth characterizing feature of the SOF approach is purely technical. The formal object one tries to build is a complete ranking of the allocations, as opposed to the definition of the best allocations. In spite of the purely technical aspect of this assumption, we believe it is of practical importance to impose it when the objective is to assess policies. Indeed, having such a ranking allows us to evaluate reforms, either by comparing a finite set of possible outcomes or by maximizing the SOF within a small set of allocations in the neighborhood of the status quo. To put it differently, real economies are far from being perfect along any ethical dimension and being able to identify directions of improvement is a legitimate objective of welfare economics.

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